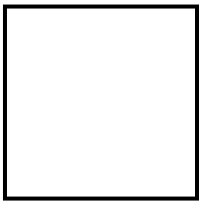
Starter
The diagram shows a square, side length x



In the table below, each row contains one value. Complete each row of this table.
Where appropriate, give answers in simplified surd form.

	x	Perimeter	Area	Diagonal
1.	**8**			
2.		**8**		
3.			**8**	
4.				**8**

An approach
Let P, A and D represent, respectively, the perimeter, area and the diagonal length of a square with side length x

So: $P = 4x$
$A = x^2$
$D = \sqrt{2x^2}$ (or $D^2 = 2x^2$)

Discussion points:
– Defining variables
– Formulating models

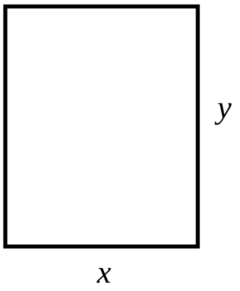
Answers

	x	P	A	D
1.	**8**	**32**	**64**	**8√2**
2.	**2**	**8**	**4**	**2√2**
3.	**2√2**	**8√2**	**8**	**4**
4.	**4√2**	**16√2**	**32**	**8**

In preparation for the main activity that follows, ask students to find a formula expressing, say, A in terms of D

Main activity: Any Two Will Do

The diagram shows a rectangle with side lengths *x* and *y*, where $x < y$



In the table below, each row contains two values. Complete each row of the table.

	x	y	Perimeter	Area	Diagonal
1.	**3**	**4**			
2.	**5**				**13**
3.			**46**	**120**	
4.			**62**		**25**
5.				**420**	**29**

An approach

Let *P*, *A* and *D* represent, respectively, the perimeter, area and the diagonal length of a rectangle with side lengths *x* and *y*

So: $P = 2(x+y)$

$A = xy$

$D = \sqrt{x^2 + y^2}$ (or $D^2 = x^2 + y^2$)

Discussion points:
– Formulating models
– Is $\sqrt{x^2+y^2}$ equal to $x+y$?
– Why is this problem harder than the starter activity?

AO3.1

1. $x = 3, y = 4$

   Direct substitution into these formulae gives $P = 14, A = 12$ and $D = 5$

2. $x = 5, D = 13$

   Using $D^2 = x^2 + y^2$ leads to $y = 12$ and then $P = 34$, $A = 60$

3. $P = 46, A = 120$

   Leads to $x + y = 23 \quad ---(1)$

   $xy = 120 \quad ---(2)$

   From (1): $y = 23 - x$

sub into (2): $x(23-x)=120$

leading to: $x^2-23x+120=0$

Use of quadratic formula (or inspired guess work!) yields

$x=8\ ,\ y=15 \quad \{\text{as } x<y\}$

and then $D=17$

Discussion point:
– Which approach is easier to work with?

Alt: Substitute $y=\dfrac{120}{x}$ into (1) to give $x+\dfrac{120}{x}=23$ which, again, leads to $x^2-23x+120=0$

4. $P=62, D=25$

Leads to

$$x+y=31 \quad ---(1)$$
$$x^2+y^2=625 \quad ---(2)$$

From (1): $y=31-x$

sub into (2): $x^2+(31-x)^2=625$

leading to: $2x^2-62x+336=0$

or $x^2-31x+168=0$

Discussion point:
– Simplifying the coefficients of an equation

Use of quadratic formula (or inspired guess work!) yields

$x=7\ ,\ y=24 \quad \{\text{as } x<y\}$

and then $A=168$

5. $A=420, D=29$

Leads to:

$$xy=420 \quad ---(1)$$
$$x^2+y^2=841 \quad ---(2)$$

From (1): $y=\dfrac{420}{x}$

Sub into (2): $x^2+\left(\dfrac{420}{x}\right)^2=841$

i.e. $x^2+\dfrac{176400}{x^2}=841$

Discussion point:
– Recognising a quadratic in another variable

leading to: $x^4 - 841x^2 + 176400 = 0$

Use of quadratic formula:

$$x^2 = \frac{841-41}{2} \qquad \{\text{as } x^2 < y^2\}$$
$$= 400$$

leading to: $x = 20$, $y = 21$ and $P = 82$

<u>*Answers*</u>

	x	y	P	A	D
1.	**3**	**4**	**14**	**12**	**5**
2.	**5**	**12**	**34**	**60**	**13**
3.	**8**	**15**	**46**	**120**	**17**
4.	**7**	**24**	**62**	**168**	**25**
5.	**20**	**21**	**82**	**420**	**29**

Taking things further
For the rectangle in Question 5, where $A = 420$ and $D = 29$, find the value of P **without** finding the value of either x or y.

An approach
Reformulate the problem symbolically;

Given that $xy = 420$ and that $x^2 + y^2 = 29^2$, find the value of $2(x+y)$ without finding the values of x and y

Obviously, it suffices to find the value of $(x+y)$

AO3.1

To this end, $(x+y)^2 = \underbrace{x^2 + y^2} + 2xy$

$$= 29^2 + 2 \times 420$$

$$= 1681$$

i.e. $x + y = \sqrt{1681}$

$$= 41$$

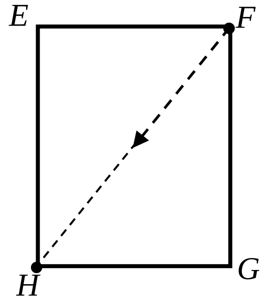
from which it follows that $P = 82$

Students could be encouraged to find a relationship connecting P, A and D
e.g. $P^2 = 4(D^2 + 2A)$, which can be used to check the answers to Questions 3. and 4.

Too abstract?

The same activity can be contextualised.

Example
A field surrounded by four roads is modelled by a rectangle *EFGH*, as shown in the diagram.



A hiker is at point *F* and wants to walk to point *H* (home)

The perimeter of the field is 46 km and its area is 120 km^2

(a) Find the length of the diagonal *FH*

(b) Give one reason why the actual distance walked from *F* to *H* may not be equal to your answer to part (a)

(c) Give another one reason why walking directly from *F* to *H* may not provide the hiker with the fastest route home.

Suggested answers

(a) requires some basic modelling, as described in the main activity.

AO3.3
AO3.4

(b) e.g. the field may not be flat.

AO3.2

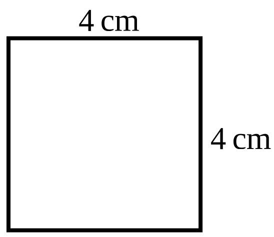
the field may not be exactly rectangular

AO3.5

(c) e.g. the field may be muddy so it might take longer to walk through the field than around its outside.

AO3.2

Finisher



In this investigation, a square is not considered to be a rectangle.

A 4cm x 4cm square has perimeter 16 cm and area 16 cm^2
So, ignoring units, the perimeter and area of this square are numerically equal.

- Find the dimensions of a rectangle with integer side lengths such that its perimeter and area are numerically equal. Can you find any more? If not, explain why.

- What if the condition ‘integer side lengths’ is dropped?